Precise determination of low-Q nucleon electromagnetic form factors and their impact on parity-violating e–p elastic scattering

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The extraction of the strangeness form factors from parity violating elastic electron-proton scattering is sensitive to the electromagnetic form factors at low Q^2. We provide parameterizations for the form factors and uncertainties, including the effects of two-photon exchange corrections to the extracted EM form factors. We study effect of the correlations between different form factors, in particular as they impact the parity violating asymmetry and the extraction of the strangeness form factors. We provide a prescription to extract the strangeness form factors from the asymmetry that provides an excellent approximation of the full two-photon correction. The corrected form factors are also appropriate as input for other low-Q analyses, although the effects of correlations and two-photon exchange corrections may be different.

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I. INTRODUCTION

The parity-violating (PV) asymmetry in elastic scattering of polarized electrons from unpolarized protons can be used to extract information on the strangeness contribution to the proton form factors [1, 2, 3]. Because the electromagnetic (EM) coupling is proportional to the quark charge-squared, scattering from the proton is strongly dominated by interaction with the up quarks. Electron–neutron scattering provides a different relative weighting of the up and down quark distributions, allowing one to study the difference between up and down quark contributions to the nucleon form factor under the assumption that the up-quark distribution in the proton is identical to the down quark distribution in the neutron, and neglecting heavier quarks. Because the parity violating cross section comes from interference between photon and Z exchange, the quark flavors have a different weighting in the interaction, allowing separation of up, down, and strange contributions to the form factors by combining proton and neutron electromagnetic form factors and parity-violating e–p scattering. However, the small contribution of the strange quarks to the parity-violating asymmetry requires precise knowledge of the contributions from the up and down quarks before one is able to achieve sensitivity to the strange quark contributions.

The parity-violating asymmetry arises due to interference between photon exchange and Z exchange, and in the Born approximation is given by [4]

$$A_{PV}^{Born} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E + A_M + A_A}{(\tau G_{Mp}^2 + \varepsilon G_{Ep}^2)}, \tag{1}$$

where G_F is the Fermi constant, α is the fine structure constant, and Q^2 is the four-momentum transfer squared. The individual asymmetry terms can be written in terms of the proton's EM vector form factors, G_{Ep} and G_{Mp}, and the proton's neutral weak vector and axial form factors, G_{Ep}^Z, G_{Mp}^Z, and G_A^Z:

$$A_E = \varepsilon G_{Ep} G_{Ep}^Z, \quad A_M = \tau G_{Mp} G_{Mp}^Z,$$
$$A_A = (1 - 4\sin^2\theta_W)\varepsilon' G_{Mp} G_A^Z,$$

where θ_e is the electron scattering angle, $\tau = Q^2/(4M_p^2)$, $\varepsilon^{-1} = (1+2(1+\tau)\tan^2\theta_e)$, θ_W is the weak mixing angle, and $\varepsilon' = \sqrt{\tau(1+\tau)(1-\varepsilon^2)}$.

In the standard model and with the assumption of isospin symmetry, the weak form factors can be expressed in terms of the proton and neutron EM form factors and the strangeness contribution to the nucleon EM form factors, G_{Es} and G_{Ms}, neglecting contributions from heavier quarks [3]. Making this substitution, and removing the common factor $A_0 = -(G_F Q^2)/(4\pi\alpha\sqrt{2})$, A_{PV} contains the terms

$$(1 - 4\sin^2\theta_W), \tag{2}$$
$$\frac{-\varepsilon G_{Ep} G_{En}}{\sigma_{\rm red}}, \tag{3}$$
$$\frac{-\tau G_{Mp} G_{Mn}}{\sigma_{\rm red}}, \tag{4}$$
$$\frac{-\varepsilon'(1 - 4\sin^2\theta_W) G_{Mp} G_A^Z}{\sigma_{\rm red}}, \tag{5}$$

which depend only on quantities that are measured or which can be reliably estimated, and one final term,

$$\frac{\varepsilon G_{Ep} G_{Es} + \tau G_{Mp} G_{Ms}}{\sigma_{\rm red}}, \tag{6}$$

which contains the unknown quantities of interest: G_{Es} and G_{Ms}. In the above expressions, we have written the denominator in terms of the e–p reduced cross section, $\sigma_{\rm red} = \tau G_{Mp}^2 + \varepsilon G_{Ep}^2$.

To extract the strangeness-containing term, the best known values and uncertainties for the other terms are needed. We present an analysis of the *world* e–p and e–n scattering data to determine the nucleon form factors, the e–p reduced cross section (the denominator of Eqs. 3-6), and their uncertainties. We also study the impact of the *correlations* between the different form factors as well as the effect of two-photon exchange (TPE) corrections on the extraction of G_{Es} and G_{Ms}. Electroweak radiative corrections have been calculated [5, 6], and their uncertainties do not generally limit the extraction of the strangeness contributions.

The extracted form factors and uncertainties are also appropriate for use in the analysis of other high precision, low Q^2 experiments. However, the analysis of correlations and TPE exchange effects presented in this paper is aimed specifically at parity-violating elastic electron-proton scattering. Care must be taken in using these fits in analysis of other experiments, as it is necessary to determine if the analysis requires the Born form factors or simply needs the form factors as a parameterization of the elastic cross section. When the cross section is required, using the Born form factors requires making an explicit correction for TPE effects [7]. For other cases, such as the extraction of the axial form factor from neutrino scattering [8], determining corrections to hyperfine splitting in hydrogen [9], or determining the Bethe-Heitler term in the analysis of DVCS measurements, one needs to carefully consider whether TPE corrections are needed and to which degree they are different from the ones needed for the unpolarized cross section.

II. ANALYSIS OF LOW-Q DATA

A. Proton form factors

A fit to the world e-p cross section data at very low momentum transfer has been described in [10]. This fit uses a Continued Fraction (CF) expansion,

$$G_{CF}(Q) = \cfrac{1}{1+\cfrac{b_1 Q^2}{1+\cfrac{b_2 Q^2}{1+\cdots}}}, \tag{7}$$

of G_{Ep} and G_{Mp} most suitable for the lower momentum transfers, and extends up to $Q = \sqrt{Q^2} \approx 0.8$ GeV/c. Note that these fits should only be used in the quoted range of Q values. The analysis includes the effects of Coulomb distortion which, contrary to common belief, are *not* negligible [11]. The effect of two-photon exchange *beyond* Coulomb distortion, which includes only the exchange of an additional soft photon, has also been studied [12, 13]. In our main analysis, we will correct the proton cross sections for Coulomb distortion, though we also provide parameterizations using the full calculation for TPE effects and discuss the impact of TPE corrections on the neutron form factors.

TABLE I: Fit parameters for the low-Q form factors, valid up to $Q = 1$ GeV/c, using the CF parametrization of Eq. 7 (with Q^2 in (GeV/c)^2) for G_{Ep}, G_{Mp}/μ_p, G_{Mn}/μ_n, and the parameterization of Eq. 8 for G_{En}. The proton data are corrected for Coulomb distortion.

	b_1	b_2	b_3	b_4	b_5
G_{Ep}	3.440	–0.178	–1.212	1.176	–0.284
G_{Mp}/μ_p	3.173	–0.314	–1.165	5.619	–1.087
G_{En}	0.977	–20.82	22.02	-	-
G_{Mn}/μ_n	3.297	–0.258	0.001	-	-

Here, we extend this fit to higher momentum transfers, up to $Q = 1.2$ GeV/c, such as to sufficiently bracket the kinematics covered by the different PV experiments. The approach taken is identical to the one employed in [10]: We start from the *world* cross sections for e–p scattering [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30], apply the Coulomb corrections according to [11, 31] and fit the cross sections with CF parameterizations of both G_{Ep} and G_{Mp}. The longitudinal/transverse (L/T)-separation is done implicitly by the fit.

This approach allows one to keep track of the random and systematic uncertainties of the data and propagate them to the final quantities of interest. From the resulting CF parameters and the error matrix of the fit one can calculate the values of G_{Ep} and G_{Mp} together with their random error for any desired value of Q. To obtain the systematic error of the proton form factors, each data set is changed by the quoted systematic uncertainty, the world set is refit, and the change in G_{Ep} and G_{Mp} calculated. These changes are added up quadratically for all data sets, yielding the systematic uncertainty on G_{Ep} and G_{Mp}. This is usually the dominant error. The total error is obtained by adding quadratically the random and the systematic errors, a procedure that should be applicable given the large number of data sets used.

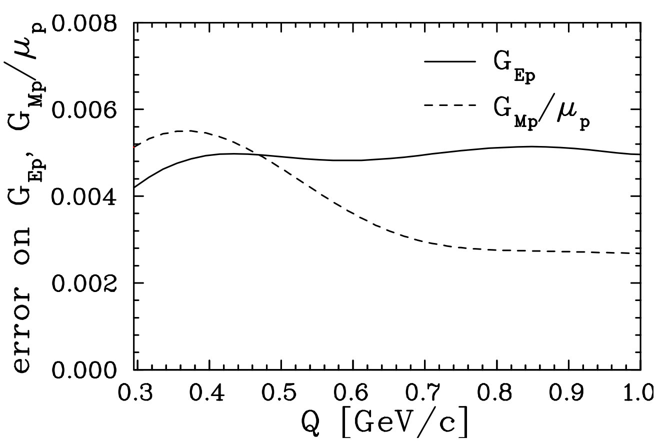

FIG. 1: Uncertainties in the fits for G_{Ep} (solid) and G_{Mp}/μ_p (dashed). The uncertainty is the random and systematic uncertainties, combined in quadrature.

The fit to the proton cross sections, after correcting for Coulomb distortion, yields the coefficients given in Table I. The fits are valid for Q from 0.3 to 1.0 GeV/c. The uncertainty for G_{Ep} (G_{Mp}/μ_p), is given by the solid (dashed) line in Fig. 1. At low Q, the error bar on G_{Mp} is larger than the one on G_{Ep} as the data base is less complete, although in the low-Q region there are two data sets with measurements at 180° [22, 26].

B. Neutron form factors

The most precise data for the neutron magnetic form factor G_{Mn} come from measurements of the ratio of ^{2}H$(e,e'n)$ to ^{2}H$(e,e'p)$ [32, 33]. The value of G_{Mn} is extracted from the neutron cross section, which is determined from the combination of the neutron to proton ratio in deuterium and the (free) proton elastic cross section. We also include measurements from the asymmetry on polarized ^{3}He [34], which are of somewhat lower precision, and data points for $Q < 1.3$ GeV/c from Ref. [35]. The high Q points have larger uncertainties, and are outside the range of validity of the fit, but are included to avoid "extreme" behavior for $Q < 1$ GeV/c. We fit these data to a 3rd order CF expansion, and the parameters are shown in Tab. I. The random and systematic uncertainties of G_{Mn} have been estimated in Ref. [32]. In the range of momentum transfer of interest here, they are approximately 1.5%, roughly independent of Q.

The value of the neutron charge form factor, G_{En}, is obtained by fitting all data presently available from polarization-transfer experiments [36, 37, 38, 39, 40, 41, 42, 43, 44]. Care has been taken to employ the most recent values, as some of the experimental G_{En}'s published early on did not contain the best corrections for FSI and MEC (or no corrections at all). To study the uncertainty due to FSI and MEC corrections, an additional uncertainty equal to 30% of the calculated correction was added in quadrature with the experimental uncertainties. Including this additional uncertainty in the extraction of G_{En} has little effect on the fit or the uncertainties. Also included in the fit are the G_{En}-values determined from the deuteron C2 form factor [45] and the slope of G_{En} at $Q = 0$, known from n–e scattering. For G_{En} the error bars of the published data contain a mix of random and systematic uncertainties. This mix is difficult to take apart, and therefore no distinction between random and systematic errors is made here.

The fits of G_{En} are done using a modified 3-parameter CF expansion:

$$G_{En}(Q) = 0.484 \cdot Q^2 \cdot G_{CF}, \qquad (8)$$

with Q^2 in (GeV/c)^2. The constant value in front fixes the slope at $Q^2 = 0$ to match the measured *rms*-radius squared value of –0.113 fm^2 [46]. The fit parameters are given in Tab. I, and the fit is valid up to $Q = 1$ GeV/c.

The error matrix is used to compute the error of G_{En} at any desired value of Q. Fits using a functional form similar to the Galster fit [47] are systematically below our fit, while fits attempting to include an explicit pion cloud contribution [48] lie above our fit. We performed fits using different fit functions and took the fit dependence as an additional contribution to the uncertainty in G_{En}. This is the dominant source of the uncertainty below $Q \approx 0.3$ GeV/c, where no direct measurements exist. The final estimated uncertainty on G_{En} is shown in Fig. 2. The figure shows the absolute uncertainty on G_{En}; the relative uncertainty is well parameterized by taking the minimum of $(5.2 + 12.6 \cdot Q^2)$%, which fits the curve below $Q = 0.3$ GeV/c, and $(9.2 + 11 \cdot \exp(-Q^2/0.19))$%, which fits above $Q = 0.3$ GeV/c.

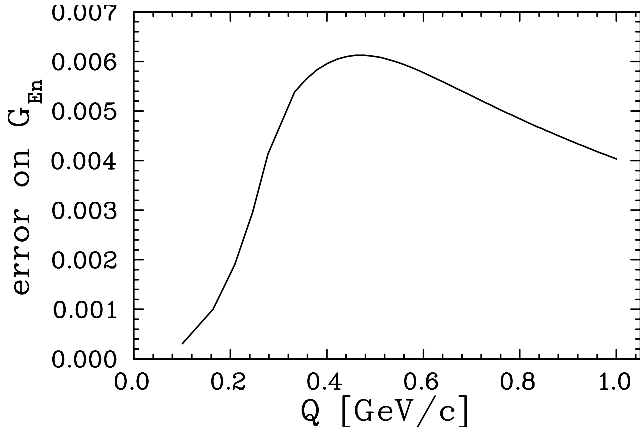

FIG. 2: Total uncertainty for the fit to G_{En}.

C. Cross Section

In the Born approximation, A_{PV} is given by Eq. 1. The inclusion of two-photon exchange terms leads to the replacement of the Born form factors $G_{Ep}(Q^2)$ and $G_{Mp}(Q^2)$ with generalized form factors that depend on both ε and Q^2, as well as introducing two new terms, A'_M and A'_A [4]. Given a complete calculation of the two-photon exchange correction, one can extract the Born form factors by correcting the Rosenbluth and polarization extractions for TPE effects, and then applying the TPE corrections to A_{PV}. However, the TPE corrections to the denominator of Eq. 1 are identical to the corrections to the e–p unpolarized cross section. So rather than correcting the unpolarized cross section measurements for TPE and then re-applying TPE correction to evaluate $\sigma_{\rm red}$, one can make a model-independent evaluation of $\sigma_{\rm red}$ by taking a fit to the TPE-uncorrected e–p cross section. Thus, we also provide a fit to the measured e–p cross section, without applying any kind of TPE correction.

The procedure is identical to the extraction of the proton form factors, except that a global fit is performed to the *uncorrected* cross sections. The reduced cross section is fit to the the form $\sigma_{\rm red} = \tau F_m(Q^2) + \varepsilon F_e(Q^2)$, such that

TABLE II: Fit parameters for the low-Q e–p cross section, neglecting TPE corrections.

	b_1	b_2	b_3	b_4	b_5
F_e	3.366	–0.189	–1.263	1.351	–0.301
F_m/μ_p	3.205	–0.318	–1.228	5.619	–1.116

in the Born approximation, $F_m = G_{Mp}$ and $F_e = G_{Ep}$. While the fit to the TPE-uncorrected data can also have an ε dependence, a global analysis of the ε dependence of σ_{ep} indicates that that deviations from linearity are extremely small [49]. Table II gives the parameters for the fit to the uncorrected cross sections. This fit is appropriate both for the reduced cross section term in the evaluation of A_{PV}, but also as a parameterization of the elastic cross section with the TPE corrections absorbed into the fit function. It is therefore useful as a low Q model of the elastic cross section if one does not wish to explicitly treat the TPE corrections for unpolarized e–p scattering.

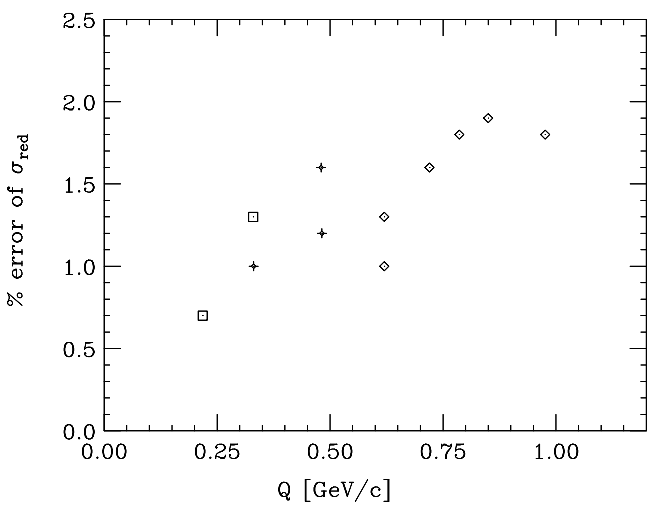

FIG. 3: Total uncertainty for the fit to the TPE-uncorrected value of $\sigma_{\rm red}$ at the kinematics of several past and planned measurements. The kinematics of forward angle are shown for the JLab ($\diamond$), while backward angle kinematics are shown for Bates ($\Box$) measurements. The Mainz ($+$) points are for forward angle measurements at low Q, and backward angle measurements at higher Q.

The uncertainties in F_e and F_m are essentially identical to those of G_{Ep} and G_{Mp}. However, the calculation of the uncertainty in $\sigma_{\rm red}$ requires special care. The values of F_e and F_m are strongly correlated as they result from the (implicitly made) L/T-separation of the cross sections. The uncertainty on the cross section thus is smaller than the one one would obtain by combining the errors in F_e and F_m. The error matrix is used to evaluate the random uncertainties, while the systematic uncertainty is taken as the combined effect of individually varying the normalization of each data set. It is not possible to provide a simple parameterization for the cross section uncertainty at all ε and Q values. The cross section uncertainties at kinematics corresponding to a variety of PV experiments are given in Figure 3. For the forward angle measurements, the cross section uncertainty is a relatively simple function of Q, so the uncertainty for other large-ε can be estimated from the small angle data in Fig. 3. The Q dependence is more complicated for large angle, and we have tried to include the complete set of planned measurements.

TABLE III: Fit parameters for the low-Q proton form factors, using the two-photon exchange correction from Ref. [12].

	b_1	b_2	b_3	b_4	b_5
G_{Ep}	3.478	–0.140	–1.311	1.128	–0.233
G_{Mp}/μ_p	3.224	–0.313	–0.868	4.278	–1.102

D. Two-photon exchange beyond Coulomb distortion

In the extractions of the proton form factors described above, we have applied Coulomb distortion corrections [11, 31] to the e–p scattering data, and no correction to the e–n data. Coulomb distortion takes into account the effect of a second soft photon, but does not include the contribution from a second hard photon. At low Q, the difference between Coulomb distortion and full TPE is small [50].

To gauge the effect of the full two-photon corrections, we perform another extraction of the proton form factors, after correcting σ_{ep} using the TPE calculation of Blunden *et al.* [12], rather than the Coulomb distortion correction. These authors calculate the contribution of the exchange of a second photon, soft or hard, restricted to the case for the intermediary state being a proton in its ground state. This calculation includes only the unexcited proton in the intermediate state; the contribution from excited intermediate states is neglected. This calculation explains most but not all of the discrepancy between Rosenbluth and Polarization measurements above Q^2=2 (GeV/c)^2, but appears to be sufficient at lower Q^2 values. A calculation including an intermediate Δ [51] also indicates that this contribution is important at Q^2=2–3 (GeV/c)^2, but provides only a small modification below 1 (GeV/c)^2.

Table III shows the results of the fit to the proton cross sections corrected for TPE, as opposed to the Coulomb distortion corrections used for the fits in Table I. For this fit, the explicit Coulomb distortion corrections done above were omitted, as the TPE corrections already contain the contribution from Coulomb distortion. The fit is valid for Q from 0.3 to 1.0 GeV/c. The uncertainties of G_{Ep} and G_{Mp} (random plus systematic added quadratically) are essentially identical to the fit with Coulomb distortion (Fig. 1). While some of the parameters in the

fits are noticeably different, the difference between correcting for Coulomb distortion and TPE on the extracted form factors is small. While the size of the Coulomb distortion corrections can be up to 3% for G_{Ep} and 1% for G_{Mp}, the difference between Coulomb and full TPE corrections is typically 0.3–0.4%, and never more than 0.5% for G_{Ep} and 0.7% for G_{Mp}. This difference is always less then the uncertainties shown in Fig. 1, and more importantly, is well within the radiative correction uncertainties assumed in the initial measurements, which are dominated by the uncertainty in TPE contributions. While the estimates of the uncertainties in the radiative correction procedure were clearly underestimates when neglecting TPE corrections, they provide a reasonable estimate of the uncertainty in the TPE calculation, especially for these small Q values.

For the neutron, the TPE correction to the cross section as calculated by Blunden *et al.* [12] is well parameterized at low Q as $\Delta\sigma/\sigma = 0.8\% \cdot Q^2 \cdot (1-\varepsilon)$, with Q^2 in $(\mathrm{GeV/c})^2$. For $Q < 1$ GeV/c,this yields a maximum correction go G_{Mn} of 0.4% at $Q = 1$ GeV/c and large scattering angle. Since most measurements are at relatively forward angle, the typical correction is $\lesssim$0.1%.

The calculated two-photon corrections to the polarization measurements of the neutron electric form factor are extremely small. It must be noted, however, that most modern experiments determining G_{En} measure an asymmetry depending on the ratio G_{En}/G_{Mn}, so a TPE correction to G_{Mn} propagates into the extracted value of G_{En}. As this correction is well below 1%, the effect is negligible compared to the experimental uncertainties of these measurements.

Thus, the overall difference between the full TPE correction and the Coulomb distortion is quite small. For protons, this different can amount to about half of the final uncertainty coming from the input form factors, but is within the radiative correction uncertainty applied in the individual measurements, and thus is properly accounted for in the final uncertainties. However, one also has to consider correlations between the uncertainties in different form factors, which can enhance the effect on the parity violating asymmetry. This is discussed in Sec. III A.

III. DETERMINATION OF A_{PV}

Given the nucleon electromagnetic form factors, as well as θ_W and G_A^Z, one can calculate $A_{PV}^{S=0}$, the PV asymmetry for $G_{Es} = G_{Ms} = 0$. One then takes the difference between the measured asymmetry and $A_{PV}^{S=0}$ as the contribution from the unknown term (Eq. 6). To obtain a reliable value for the strangeness contribution, one must include radiative corrections in evaluating $A_{PV}^{S=0}$, and determine the uncertainty in $A_{PV}^{S=0}$ due to uncertainty in the form factor and other terms. The uncertainty in $A_{PV}^{S=0}$ is usually determined by varying the individual form factors that contribute to the asymmetry by their assumed uncertainties. This approach ignores two effects which could be significant in these measurements. First, it neglects the correlation between the extracted values of the electromagnetic form factors, which can impact the total uncertainty on $A_{PV}^{S=0}$. Second, it neglects the impact of two-photon exchange corrections on $A_{PV}^{S=0}$, as well as their effect on the extracted values of the form factors. In the following sections, we will study the effect of the correlated uncertainties between the extractions of the different form factors, and estimate the size of TPE corrections. We will present a procedure for determining the size and uncertainty of the parity-violating asymmetry that does not require an explicit calculation of TPE, but which minimizes the uncertainty in extracting the strangeness contributions. We will compare this to the result obtained if one ignores both TPE corrections to A_{PV} and the TPE corrections in the extraction of the electromagnetic form factors, as has been done in all previous extractions of the strangeness contributions.

A. Impact of correlations

The effects of these correlations need to be evaluated to obtain an accurate measure of the uncertainty in $A_{PV}^{S=0}$. Taking the correlations into account can noticeably increase or decrease the contribution of the form factors to the total uncertainty. We examine here the impact of these correlations on the evaluation of $A_{PV}^{S=0}$, in the region of $0.3 < Q < 1.0$ GeV/c, where such measurements have been carried out or proposed.

At the Q values of interest here the main issues of concern are the anti-correlation between G_{Ep} and G_{Mp} extracted from Rosenbluth separation, the correlation between G_{Mn} and σ_{ep} (σ_{red}) for data extracted in measurements of the proton/neutron ratio or the ^{3}He quasielastic asymmetry, and the correlation between G_{En} and G_{Mn} in polarization measurements that extract G_{En}/G_{Mn}. In some cases, it is difficult to precisely quantify the level of correlation and difficult to propagate to the value of $A_{PV}^{S=0}$. The aim of this analysis is to determine where these correlations can be neglected or treated in some approximate fashion, and to determine the uncertainty related to these approximations.

1. *Correlation between G_{Ep} and G_{Mp}*

At the low values of Q of interest here, the proton form factors are determined by L-T separations, which yield a significant anti-correlation between the extracted values of G_{Ep} and G_{Mp}. This has an impact in determining the uncertainty in σ_{red}, which appears in the denominator of most of the terms, as well as introducing a correlation between the errors in the term involving G_{Ep} and the term involving G_{Mp}.

The largest effect results from the fact that the uncertainty on σ_{red} is much smaller than one obtains by

varying G_{Ep} and G_{Mp} individually. By treating $\sigma_{\rm red}$ and $\delta\sigma_{\rm red}$ as being independent quantities from G_{Ep} and G_{Mp}, we eliminate the overestimate of the uncertainty, and we better account for TPE effects as well (see Sec. II C). When $\sigma_{\rm red}$ is extracted directly from the cross sections, the remaining correlation between $\sigma_{\rm red}$ and the individual form factors is very small.

The remaining effect is the correlation between the terms in A_{PV} involving G_{Ep} and G_{Mp} (Eqs. 3 and 4). These terms have the opposite sign in the final asymmetry, so the anti-correlation between the values of G_{Ep} and G_{Mp} will tend to increase the total uncertainty. For small ε values, the term involving G_{Ep} is only a few percent of the total asymmetry, and so its uncertainty has a negligible small effect ($< 0.1\%$ of $A_{PV}^{S=0}$ for $\varepsilon < 0.05$). At large ε values, this term contributes roughly 20% the of $A_{PV}^{S=0}$, and so has a greater impact. If taken to be 100% anti-correlated with G_{Mp}, the difference at large ε grows from 0.1% of $A_{PV}^{S=0}$ at $Q^2 = 0.1$ (GeV/c)^2 to 0.4% at 1 (GeV/c)^2. In reality, the effect will be smaller, as the correlation is not 100%, especially in the region where G_{Mp} is mainly given by the data taken at 180°. The completed and proposed measurements of $A_{PV}^{S=0}$ typically have ∼10% precision on $A_{PV}^{S=0}$, and never better than 4%, and so neglecting this correlation will again have a very small effect on the final uncertainty. Thus, it is a good approximation to treat G_{Ep} and G_{Mp} as uncorrelated, as long as one takes the uncertainty $\delta\sigma_{\rm red}$ directly, rather than calculating $\delta\sigma_{\rm red}$ from δG_{Ep} and δG_{Mp}.

2. *Correlation between G_{Mn} and $\sigma_{\rm red}$*

The most precise values of G_{Mn} come from measurements of the ratio of d(e,e'n) to d(e,e'p). By normalizing to σ_{ep}, measured on the proton, one can extract σ_{en} and thus G_{Mn}, since the contribution from G_{En} is almost negligible. Measurements utilizing quasielastic scattering of polarized electrons from polarized ^{3}He are essentially measuring the same quantity [34]. The transverse asymmetry for scattering from the polarized neutron is nearly independent of the neutron form factors, and so the ^{3}He asymmetry is mainly sensitive to the dilution due to the two (nearly) unpolarized protons, and thus is sensitive to σ_{ep}/σ_{en}. Therefore, both experiments yield a direct correlation between the extracted value of G_{Mn} and the value of σ_{ep} used in the analysis.

Because of this correlation, it is important that TPE are treated in a consistent fashion. If the proton form factors are corrected for TPE, then the TPE contributions must be included in calculating σ_{ep} as observed in the σ_{en}/σ_{ep} measurements. Because TPE corrections were neglected in both the extraction of the proton form factors and the calculation of σ_{ep} as used in the G_{Mn} extractions, one obtains a correct parameterization of the unpolarized e–p cross section, as in Sec. II C.

The typical uncertainties in σ_{ep} at kinematics where G_{Mn} has been extracted are ≈1.4%, yielding a contribution to the uncertainty in G_{Mn} of ≈0.7%. For $Q < 1$ GeV/c, the uncertainty in $A_{PV}^{S=0}$ due to G_{Mn} is close to half the size of the uncertainty coming from $\sigma_{\rm red}$, so a perfect correlation would have the effect of reducing this contribution to the uncertainty by roughly a factor of two. However, the uncertainty on $\sigma_{\rm red}$ is usually not the dominant contribution, and so the effect of reducing this contribution is never more than 0.4% of $A_{PV}^{S=0}$. In fact, the effect is even smaller since the uncertainties are not 100% correlated between different extractions of G_{Mn}.

3. *Correlation between G_{En} and G_{Mn}*

The polarization measurements are sensitive only to the ratio G_{En}/G_{Mn}, and thus the error in G_{Mn} used in the analysis yields an identical shift in G_{En}. However, the contribution to δG_{En} from δG_{Mn} is a very small part (typically 10%) of the total uncertainty in G_{En}.

B. Two-photon exchange corrections to A_{PV}

A full calculation of the TPE corrections to A_{PV} requires starting with the TPE-corrected (i.e. Born) form factors, and then applying the full TPE corrections for parity-violating scattering to Eq. 1. We have made fits to the proton and neutron form factors, both with partial TPE corrections, neglecting the effect of a second hard photon, and with full TPE corrections. The full corrections are more model dependent, but are close to the corrections applied, as discussed in Sec. II D. The uncertainties assumed for the radiative corrections are now consistent with the corrections applied, even for the partial correction.

We apply TPE corrections based on the formalism by Afanasev and Carlson [4]. This includes the effect of the two photon box (and crossed-box) diagrams, but not the effect of the γ-Z box diagram, which has been examined (for $Q^2 = 0$) in Ref. [52]. It should be noted that for these corrections, it is *not* sufficient to apply the correction for only the second soft photon; one must go beyond Coulomb distortion. This is because the Coulomb distortion is a long range contribution that, to first order, yields a helicity-independent rescaling of the cross sections, and thus cancels in the evaluation of A_{PV}. So for A_{PV}, one must use the full calculation, including the exchange of a hard photon.

For convenience, we separate the TPE effects into three categories. First, there are two new terms, A'_M and A'_A, that appear in the expression for A_{PV}. Second, the Born form factors that go into the terms A_E, A_M, and A_A in Eq. 1 are replaced with generalized form factors that depend on both ε and Q^2. Finally, the TPE correction changes the unpolarized e–p reduced cross section, $\sigma_{\rm red}$, which appears as part of the denominator. This involves both replacing the Born form factors with the generalized

form factors, and introducing new terms related the new amplitude that appears when including TPE.

We evaluate the corrections to A_{PV} using the TPE calculations of Refs. [12, 13], which give identical results for the Q range of interest. The new terms, A'_M and A'_A, have small contributions, below 0.1% to A_{PV} for $Q < 1$ GeV/c. While these terms become comparable in size to the corrections to A_A, A_E, and A_M at higher Q values, they are negligible at low Q. The effect of the generalized form factors on A_E, A_M, and A_A is larger, but still relatively small; on the order of 1% for $Q < 1$ GeV/c. The final TPE contribution, the correction to $\sigma_{\rm red}$ (the denominator of Eq. 1) can be 2–3% at small ε values. Due to cancellation between the different terms, the combined effect of TPE on A_{PV} is $\lesssim$1% for $Q < 1$ GeV/c.

However, the TPE correction to the A_{PV}^{Born} is not the complete story; one must also take into account the *indirect* impact of TPE corrections on the extraction of $A_{PV}^{S=0}$. In the past, TPE contributions were neglected in extracting the nucleon electromagnetic form factors, and so the calculated value of A_{PV} is *not* the correct value for A_{PV}^{Born}. The TPE corrections to G_{Ep} and G_{Mp} change $A_{PV}^{S=0}$ by 1–2% for Q below 1 GeV/c, and as much as 5% for $Q = 2$ GeV/c. The largest corrections coming for small scattering angles, where the precision of the completed and planned measurements is the highest, and so one must apply TPE correction to the extraction of the nucleon electromagnetic form factors, as done in Section II A.

In summary, the largest effects are the corrections to the Rosenbluth extracted values of G_{Ep} and G_{Mp}, and the application of the TPE effects to the denominator of Eq. 1. The TPE effect on the denominator are identical to those in the unpolarized cross section measurements, and so a model-independent extraction of the denominator can be achieved by using *uncorrected e–p* cross section. We have provided TPE-corrected fits to the form factors, as well as uncorrected fits to the unpolarized cross sections, which allow these corrections to be applied without requiring an explicit calculation of the TPE corrections to $A_{PV}^{S=0}$. If one neglects the remaining corrections to the numerator of Eq. 1, the result is within 1% of the full calculation. However, a simple linear parameterization of these remaining terms provides a calculation of $A_{PV}^{S=0}$ that is within 0.2% of the full calculation:

$$A_{PV} \rightarrow A_{PV} \cdot [1 + (C_0 + \varepsilon C_1)] \qquad (9)$$

where $C_0 = .013 - .022Q$, $C_1 = -.010 + .018Q$, with Q in GeV/c.

Figure 4 compares various approximations for A_{PV} to the full calculation explicitly including TPE corrections. The top left panel shows that the correction to the Born value is small, due to the relatively small direct TPE contributions, and the cancellation between TPE contributions to different terms. The bottom left plot shows the error made when neglecting TPE corrections in both the calculation of A_{PV} *and* the extraction of the EM form factors. The right hand plots show the approximation discussed in this paper, neglecting the additional correction due to the effect on the numerator in Eq. 1 (top figure), and including the parameterization of this correction from Eq. 9 (bottom figure).

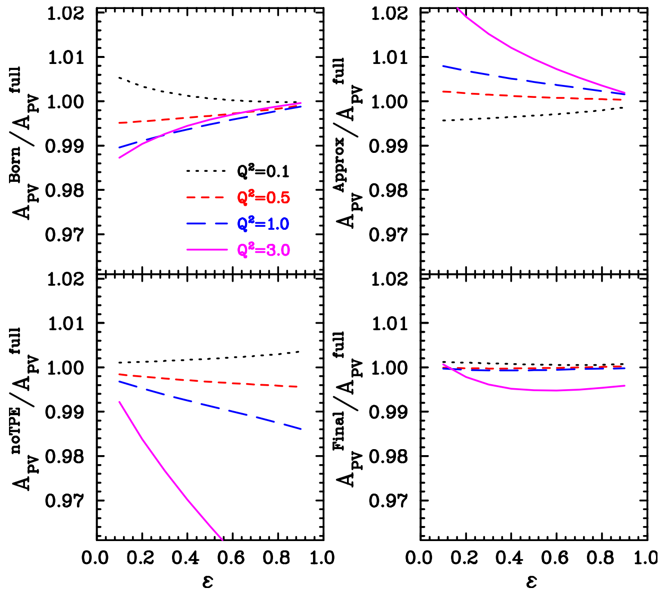

FIG. 4: Comparisons of different calculations of A_{PV} to the calculation including the full TPE effects. Top left plot is A_{PV}^{Born}, bottom left is neglecting TPE in both the extraction of the EM form factors and in calculating A_{PV}, i.e. the procedure used in analyzing previous experimental results. The top right is the approximation presented here, *neglecting* the additional parameterization of the TPE effect on the numerator of Eq. 1, and the bottom right is the final prescription, including this correction (Eq. 9)

C. Extension to larger Q

While corrections to individual terms in Eq. 1 can be at the 1–2% level, and additional corrections due to TPE effects in the extraction of the Born EM form factors can be even larger, significant cancellation between different terms yields a total correction that is typically below 1% for $Q < 1$ GeV/c. After applying the TPE corrections as discussed above, the uncertainties in the TPE corrections for $Q < 1$ GeV/c are dominated by the uncertainty in extracting the TPE-corrected form factors. This uncertainty is taken into account in the typical 1.5% uncertainty assumed for radiative corrections, and thus no additional uncertainty need be applied.

At larger Q, these corrections grow significantly, as shown in Fig. 4, and the total error made in neglecting TPE corrections can reach 10% by $Q = 2$ GeV/c. The procedure described here is provides a correction good to 0.2% up to $Q = 1$ GeV/c, and 1% up to 2 GeV/c. At higher Q, the corrections become even larger, and

the calculation of TPE corrections becomes less reliable. An estimate of the contributions from an intermediate Δ in the box diagram [51] indicates that this contribution is less than 0.3% for $Q^2 < 1$ (GeV/c)^2, while at $Q^2 = 3$ (GeV/c)^2, the contribution is as large as 2%, and is significantly more model dependent.

IV. SUMMARY OF THE PROCEDURE

The final prescription involves evaluating the terms in Eq. 1, using TPE-corrected fits for the nucleon form factors in the terms A_E, A_M, and A_A, the *TPE-uncorrected* fits to $\sigma_{\rm red}$ for the denominator, and applying the correction from Eq. 9 to account for the TPE corrections for the terms in the numerator of Eq. 1 and the additional terms A'_M and A'_A [4]. Without this final correction, the approximation is valid to better than 1% for Q values from 0.3-1.0 GeV/c, and better than 0.5% except for $Q \approx 1$ GeV/c and $\varepsilon < 0.5$. With this correction, the approximation is good to 0.2%.

To get the overall error of the term Eq. 6, one should quadratically add the following contributions:
– the effect of the error of G_{Ep} (Fig. 1)
– the effect of the error of G_{Mp} (Fig. 1)
– the effect of the error of G_{En} (Fig. 2)
– the effect of the error of G_{Mn} (1.5%)
– the effect of the error of the e–p cross section (the denominator of Eqs. 1-6) (Fig. 3)
– the uncertainty associated with neglected TPE corrections

Note that in evaluating the error due to G_{Ep} and G_{Mp}, the values of the form factors are changed only in the numerator of Eq. 1; the value of $\sigma_{\rm red}$ is left unchanged, as it's contribution to the uncertainty is treated separately (Sec. II C). For the complete analysis of the uncertainly of PV experiments, one must of course add the uncertainties stemming from uncertainty in θ_W and G_A^Z as well as uncertainty in the scattering kinematics.

Finally, one obtains the term involving the strange form factors by equating the term in Eq. 6 with $(A_{PV} - A_{PV}^{S=0})$. Thus, the uncertainty in $\sigma_{\rm red}$ enters again when isolating the linear combination $G_{Es} + \eta G_{Ms}$. Because $A_{PV} \approx A_{PV}^{S=0}$, the 1–2% overall scale uncertainty on the extracted value of $G_{Es} + \eta G_{Ms}$ will always be very small compared to the effect of the uncertainty of $\sigma_{\rm red}$ on $A_{PV}^{S=0}$, and so again these uncertainties can be treated as uncorrelated without significant effect on the final uncertainties.

V. CONCLUSIONS

We have evaluated the effect of TPE corrections on parity-violating elastic electron–proton scattering using the TPE exchange calculations of Refs. [12, 13]. The direct effect of TPE on the parity violating asymmetry is small, $\lesssim$1% for $Q < 1$ GeV/c. However, the effect of TPE on the Rosenbluth extractions of G_{Ep} and G_{Mp}, which are needed to extract the strangeness contribution from the asymmetry, can be significant, and should be taken into account in the analysis of the parity-violating measurements. We have provided fits to the form factors and their uncertainties, and provided a prescription to allows for an extraction of the strangeness form factors without explicitly requiring a calculation of the TPE exchange effects. As we have shown, this prescription provides an excellent approximation to the full procedure, based on tests performed using the full TPE calculation. This provides a common set of form factors and uncertainties for the analysis of low-Q parity violating measurements, as well as a consistent application of TPE corrections on the extraction of the strangeness form factors. This approach has the advantage that it can be applied without requiring an explicit calculation of the TPE amplitudes, while providing an approximation of the the TPE corrections to better than 0.2%, with model dependence in the TPE correction that is consistent with the assumed uncertainties due to RC for the measurements.

These TPE-corrected form factors are needed for determining the value and uncertainty in A_{PV} for the case of no strange quark contributions. They are also the true, Born form factors that are related to the structure of the nucleon, and which should be used in the analysis of other experiments. However, the effect of TPE is different in different observables, and one must consider if the Born form factors are the correct input in the case being considered. For example, many analyses such as Rosenbluth separations in quasielastic A(e,e'p) scattering [53] or the extraction of G_{Mn} from polarization [34] or ratio measurements [32], require knowledge of the e–p cross section to extract information on other quantities. If one uses the TPE-corrected form factors, then one must include TPE corrections in calculating the cross section. In such cases, it is simpler and less model-dependent to use the fits to the uncorrected cross sections. Other cases, such as quasielastic neutrino scattering or the case of parity-violating electron scattering considered here, will not have the same TPE effects and need to be evaluated with care.

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